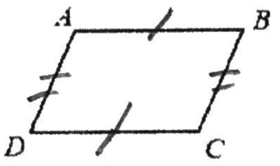
10/29/18

Proving Parallelograms

When working on a coordinate plane

METHOD 1

Prove **both** pairs of opposite sides are **congruent**.



If $\overline{AB} \cong \overline{DC}$

and $\overline{AD} \cong \overline{BC}$, then

ABCD is a parallelogram.

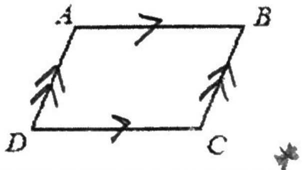
Use...

Distance formula (4 times)

$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

METHOD 2

Prove **both pairs of opposite sides** are **parallel**.



If $\overline{AB} \parallel \overline{DC}$

and $\overline{AD} \parallel \overline{BC}$, then

ABCD is a parallelogram.

*lines are parallel if their slopes are ≅

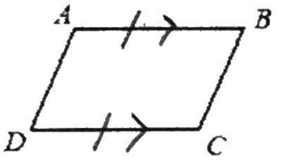
Use...

Slope formula (4 times)

$m = \frac{rise}{run} = \frac{y_2-y_1}{x_2-x_1}$

METHOD 3

Prove **one pair of opposite sides** are **congruent and parallel**.



If $\overline{AB} \parallel \overline{DC}$

and $\overline{AB} \cong \overline{DC}$, then

ABCD is a parallelogram.

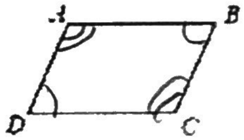
Use...

- Distance formula
- Slope formula

*for one pair of opp. sides

METHOD 4

Prove **both pairs of opposite angles** are **congruent**.



If ∠B ≅ ∠D and

∠A ≅ ∠C, then

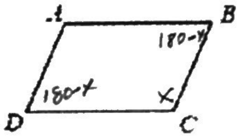
ABCD is a parallelogram.

Use...

✗

METHOD 5

Prove **two sets of consecutive angles** are **supplementary**.



If ∠C + ∠B = 180° and

∠C + ∠D = 180°, then

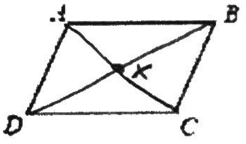
ABCD is a parallelogram.

Use...

✗

METHOD 6

Prove that **both diagonals** are **bisected**.



If $\overline{AX} \cong \overline{XC}$ and

$\overline{DX} \cong \overline{XB}$, then

ABCD is a parallelogram.

Use...

midpoint formula

$\left(\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2}\right)$

$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$ $m = \frac{y_2-y_1}{x_2-x_1} = \frac{rise}{run}$

DIRECTIONS: Determine whether the figure is a parallelogram using the distance formula.

1. $A(-7, 4)$, $B(1, 2)$, $C(9, -8)$, $D(1, -6)$

$d_{AB} = \overset{(x_1, y_1)\ (x_2, y_2)}{\sqrt{(1+7)^2 + (2-4)^2}}$

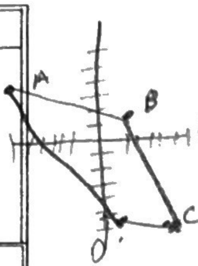
$= \sqrt{8^2 + (-2)^2}$

$= \sqrt{64+4} = \sqrt{68}$

$d_{\overline{AB}} = \sqrt{68}$ $d_{\overline{DO}} = \sqrt{68}$

$\boxed{YES}$ $d_{\overline{AO}} = \sqrt{164}$ $d_{\overline{BC}} = \sqrt{164}$

b/c 2 sets of opp. sides are ≅



DIRECTIONS: Determine whether the figure is a parallelogram using the slope formula.

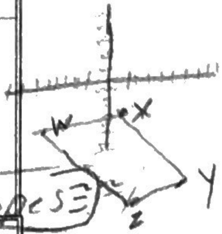
3. $W(-7, -4)$, $X(1, -6)$, $Y(5, -13)$, $Z(1, -12)$

$m_{WX} = \frac{-6+4}{1+7} = \frac{-2}{8} = \frac{-1}{4}$ with (x_1, y_1) (x_2, y_2)

$m_{\overline{NX}} = -\frac{1}{4}$ $m_{\overline{YZ}} = -\frac{1}{4}$

$\boxed{NO}$ $m_{\overline{XY}} = \frac{-7}{4}$ $m_{\overline{WZ}} = -1$

b/c there is only 1 set of opp slopes ≅



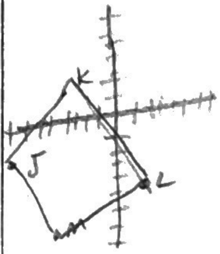
DIRECTIONS: Determine whether the figure is a parallelogram using the distance and slope formulas.

5. $J(-9, -2)$, $K(-5, 1)$, $L(1, -4)$, $M(-3, -7)$

$m_{JK} = \frac{3}{4}$ $m_{\overline{ML}} = \frac{3}{4}$

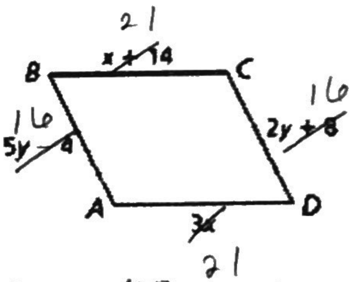
yes b/c one set of opp sides ≅ and ∥ proves a ▱.

$d_{\overline{JK}} = 5$ $d_{\overline{ML}} = 5$



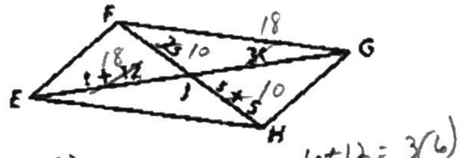
▱ - means parallelogram

Show that *ABCD* is a parallelogram for $x = 7$ and $y = 4$.



ABCD is a ▱ b/c 2 sets of opp sides ≅ proves ▱

Show that *EFGH* is a parallelogram for $s = 5$ and $t = 6$.



$2(5) = 5+5$

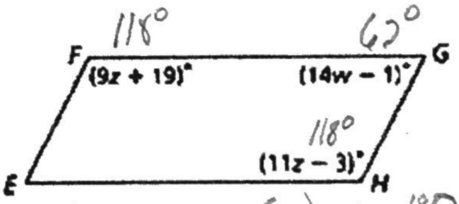
$10 = 10$

$6+12 = 3(6)$

$18 = 18$

EFGH is a ▱ b/c both diagonals bisecting → ▱

Show that *EFGH* is a parallelogram for $z = 11$ and $w = 4.5$.

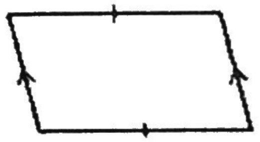


$9(11) + 19 + 14(4.5) - 1 = 180$

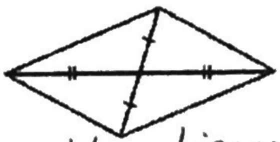
$180 = 180$ ✓

EFGH is a ▱ b/c consecutive ∠'s are supp.

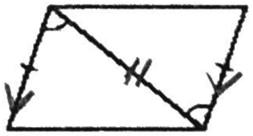
Determine if the following quadrilaterals must be parallelograms. Justify your answer.



NOT a ▱

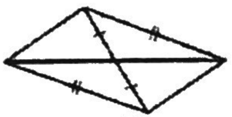


Yes - b/c diagonals bisecting → ▱



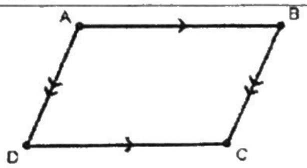
yes b/c one set of opp sides ∥ and ≅ → ▱

*converse of alt. int ∠'s



Not a ▱

Yes b/c opp. ∠'s ≅ → ▱



yes b/c opp sides ∥ → ▱